University Kasdi Merbah Ouargla

3rd year engineering

# MODULE: FUNDAMENTALS OF AI

## Lesson: Game Theory

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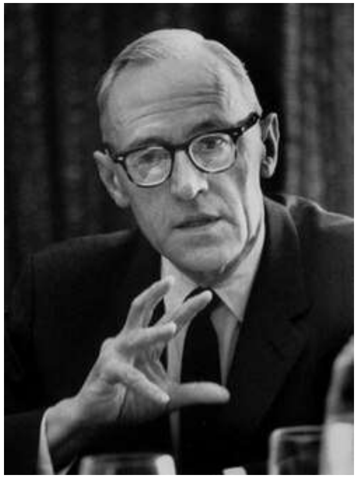
# HISTORY:

Game theory came in to existence in 20th Century.
In 1944 the mathematicians John Von Neumann and economist Oscar Morgenstern published a book Theory of game and Economic Behaviour, in which they discussed how businesses of all types may use this technique to determine the best strategies given a competitive business environment.



# DEFINITION :

In business and economics literature, the term 'game' refers to a situation of conflict and competition in which two or more competitor(or participant) are involved in the decision making process in anticipation of certain outcome over a period of time.

# WHAT IS GAME THEORY

- ***Game Theory*** is a set of tools and techniques for decisions under uncertainty involving two or more intelligent opponents in which each opponent aspires to optimize his own decision at the expense of the other opponents. In game theory, an opponent is referred to as player.
- Each player has a number of choices, finite or infinite, called strategies.

# BASIC TERMS USED IN GAME THOERY

- **PLAYERS: WHO IS INTERACTING?**
- **STRATEGIES: WHAT ARE THEIR OPTIONS?**
- **PAYOFFS: WHAT ARE THEIR INCENTIVES?**
- **INFORMATION: WHAT DO THEY KNOW?**
- **RATIONALITY: HOW DO THEY THINK?**

# BASIC TERMS USED IN GAME THOERY

1. **Player:** Each participant playing a game is called a player
2. **Pay-off:** The quantitative measure of achievement at the end of game is known as pay-off
3. **Pay-off matrix:** the table which show the outcome of the game when different strategies are adopted by different players s known as pay-off matrix
4. **Strategy:** It is the predetermined rule by which a player decides his course of action from his available courses of action
5. **Saddle Point:** the Game value is called the saddle point in which each player has a pure strategy. If saddle point exists, the game is said to be stable. It is the number which lowest in its rows and highest in columns

# BASIC TERMS USED IN GAME THOERY

**6. Value of game:** The maximum guaranteed expected outcome per play when players follow their optimal strategy is called the "value of the game". It is denoted by V. If the value of the game is zero then it is also called fair game.

**7. Maxmini:** Maximize the minimum value of gain (Max)

**8. Minimax:** Minimize the maximum value of loss (Min)

**9. Strictly determinable game:** A game is said to be strictly determinable if the maximum value is equal to minimax value but not zero

# CLASSIFICATION OF GAMES

- **Two-Person Game** – A game with 2 number of players.
- **N-Person Game** – A game with number more than two players.

- **Zero-Sum Game** – A game in which sum of amounts won by all winners is equal to sum of amounts lost by all losers.
- **Non-Zero Sum Game** – A game in which the sum of gains and losses are not equal.

- **Pure-Strategy Game** – A game in which the best strategy for each player is to play one strategy throughout the game.
- **Mixed-Strategy Game** – A game in which each player employs different strategies at different times in the game.

  - **Cooperative Game:**
  - **Non-cooperative game:**

# MATHEMATICAL FORMULATION

Let player A have 'm' courses of action and player B has 'n' courses of action. The game can be shown by a pair of matrixes constructed as shown:

- Raw represents for each matrix are the course of actions available to player A
- Columns represents for each matrix are the courses of actions available to player B
- The cell values represents payments to A in case of player A's pay-off matrix
- The cell values represents payments to B in case of player B's pay-off matrix
- Player A is called maximizing player and he will always try to maximize the minimum gain
- Player B is called minimizing player and he will always try to minimize the maximum loss.
- The sum of payoff matrix for A and B is a null matrix

Player B

| | 1 | 2 | 3 | … | N |
|---|---|---|---|---|---|
| 1 | $a_{11}$ | $a_{12}$ | $a_{13}$ | … | $a_{1n}$ |
| 2 | $a_{21}$ | $a_{22}$ | $a_{23}$ | … | $a_{2n}$ |
| 3 | $a_{31}$ | $a_{32}$ | $a_{33}$ | … | $a_{3n}$ |
| … | … | … | … | … | … |
| M | $a_{m1}$ | $a_{m2}$ | $a_{m3}$ | … | $a_{mn}$ |

Player A

Player A's Pay-off matrix

| | Player B | | | | |
|---|---|---|---|---|---|
| | 1 | 2 | 3 | … | N |
| 1 | $-a_{11}$ | $-a_{12}$ | $-a_{13}$ | … | $-a_{1n}$ |
| 2 | $-a_{21}$ | $-a_{22}$ | $-a_{23}$ | … | $-a_{2n}$ |
| 3 | $-a_{31}$ | $-a_{32}$ | $-a_{33}$ | … | $-a_{3n}$ |
| … | … | … | … | … | … |
| M | $-a_{m1}$ | $-a_{m2}$ | $-a_{m3}$ | … | $-a_{mn}$ |

Player A

Player B's Pay-off matrix

# SOLUTIONS OF THE GAMES

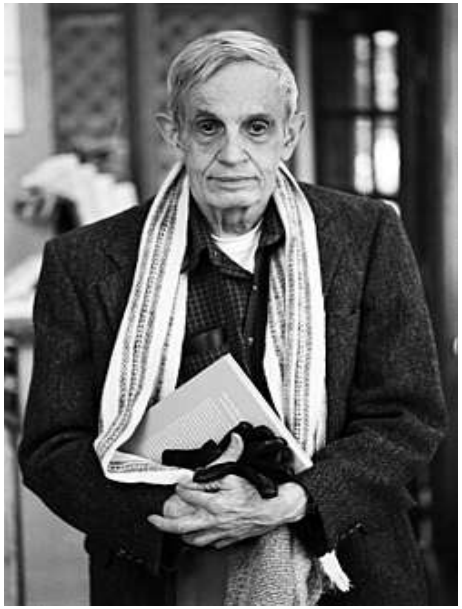
- **To predict what will be the solution/outcome of the game we need some tools:**

- **Nash equilibrium**
- **dominated strategy**
- **dominant strategy**

# Nash Equilibrium

- The decisions of the players are a Nash Equilibrium if no individual prefers a different choice.

- In other words, each player is choosing the best strategy, given the strategies chosen by the other players.



# EXAMPLE: Nash Equilibrium

Two firms, A and B, choose either **High (H)** or **Low (L)** price.
Payoffs are *(A's profit, B's profit)*.

| | B: H | B: L |
|---|---|---|
| A: H | (3, 3) | (1, 4) |
| A: L | (4, 1) | (2, 2) |

**Best responses**

- If B plays **H**, A prefers **L** (4 > 3).
- If B plays **L**, A prefers **L** (2 > 1).
- If A plays **H**, B prefers **L** (4 > 3).
- If A plays **L**, B prefers **L** (2 > 1).

**Intersection of best responses:** (**L, L**) → **Nash equilibrium** with payoffs **(2, 2)**.

No player can improve by unilaterally deviating.

*Note:* (**H, H**) yields higher joint payoff (3,3) but is not stable

# Dominant Strategies

- Dominant strategy (for a player):

A strategy is dominant if, regardless of the choices of other players, it gives that player a payoff at least as high as all their other strategies, and strictly higher in at least one case.

- To test dominance, compare line by line (or column by column) the gains of a player while staring at the opponent.

# **EXAMPLE:** Dominant strategy

Two players choose to **Cooperate (C)** or **Defect (D)**. Payoffs are *(Player A, Player B)*.

| | B: C | B: D |
|---|---|---|
| A: C | (3, 3) | (0, 4) |
| A: D | (4, 0) | (1, 1) |

**Why D is a dominant strategy for both players**

•If **B plays C**:
A gets 3 with **C** vs **4 with D** → A prefers **D.**

•If **B plays D**:
A gets 0 with **C** vs **1 with D** → A prefers **D.**

So **A's D** beats (or ties and sometimes beats) **A's C no matter what B does.**

By symmetry, the same is true for **B.**

**Result:** each player's dominant strategy is **D,** so the outcome is **(D, D)** with payoffs (1,1).

# Dominated Strategies

- An alternative that yields a lower payoff than some other strategies
- a strategy is dominated if it is always better to play some other strategy, regardless of what opponents may do
- It simplifies the game because they are options available to players which may be safely discarded as a result of being strictly inferior to other options.

# **EXAMPLE:** Dominated strategy

Payoffs are (A,B).

| | B: L | B: R |
|---|---|---|
| A: U | (3, 2) | (2, 1) |
| A: D | (1, 1) | (0, 0) |

Compare A's rows:

•If B=L: 3>13 > 13>1 (U better than D)

•If B=R: 2>02 > 02>0 (U better than D)

So **A: D** is **strictly dominated** by **A: U**. A should never play **D.**

Compare B's columns:

•If A=U: 2>12 > 12>1 (L better than R for B)

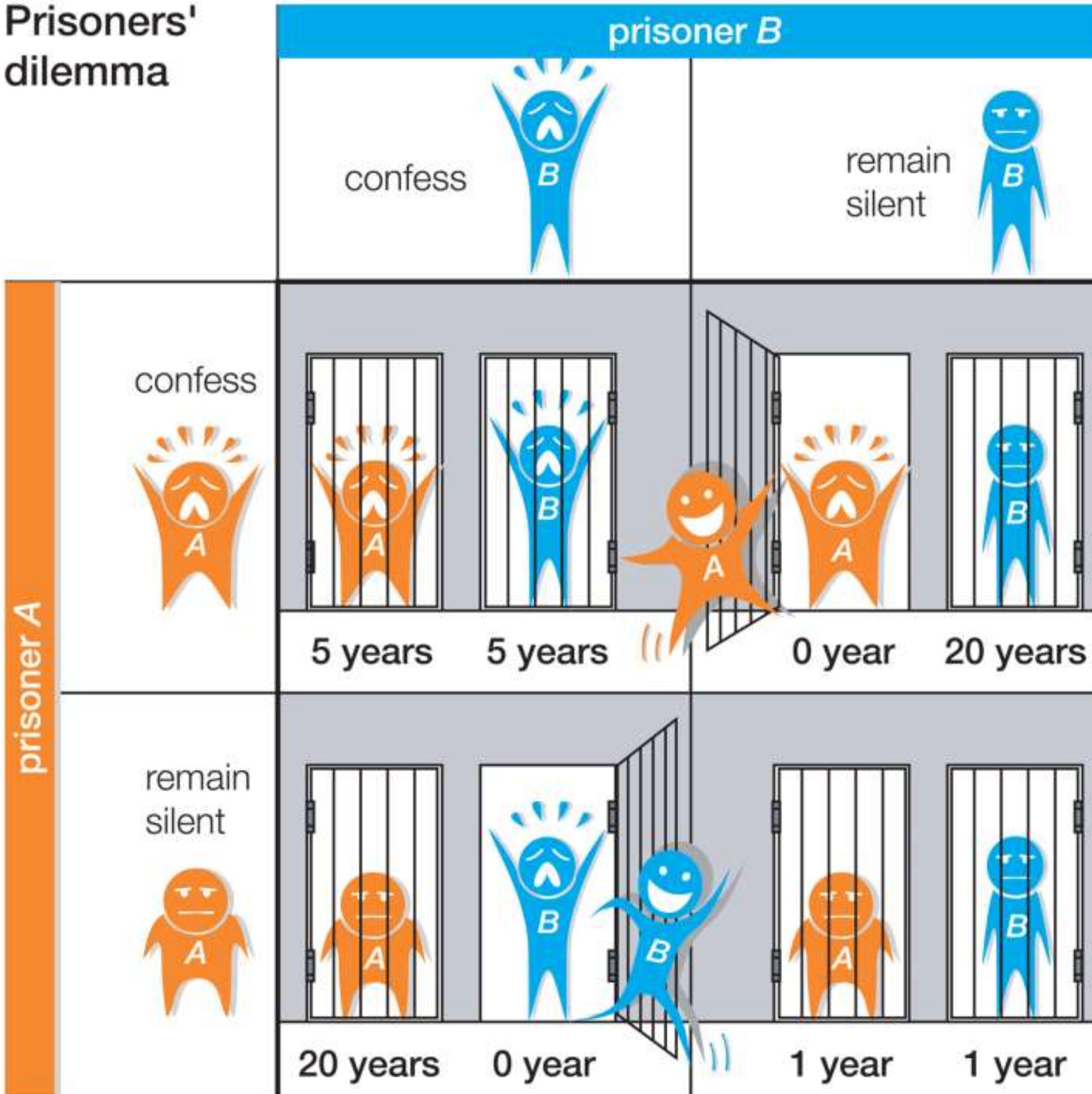
•If A=D: 1>01 > 01>0 (L better than R)

So **B: R** is **strictly dominated** by **B: L.** B should never play **R.**

# PRISONERS DILEMMA

They are in separate cells being sweated by the cops.

What do they do?



# PRISONER'S DILEMMA

The **goal of the Prisoner's Dilemma** is to **illustrate the tension between individual rationality and collective welfare**

**Why it matters:**

- Shows how **incentives** can produce socially bad outcomes.
- Motivates **mechanism design, contracts**, and **enforcement** to make cooperation rational.
- In **repeated** versions, it helps study how **trust, reputation, and punishment** can sustain cooperation.

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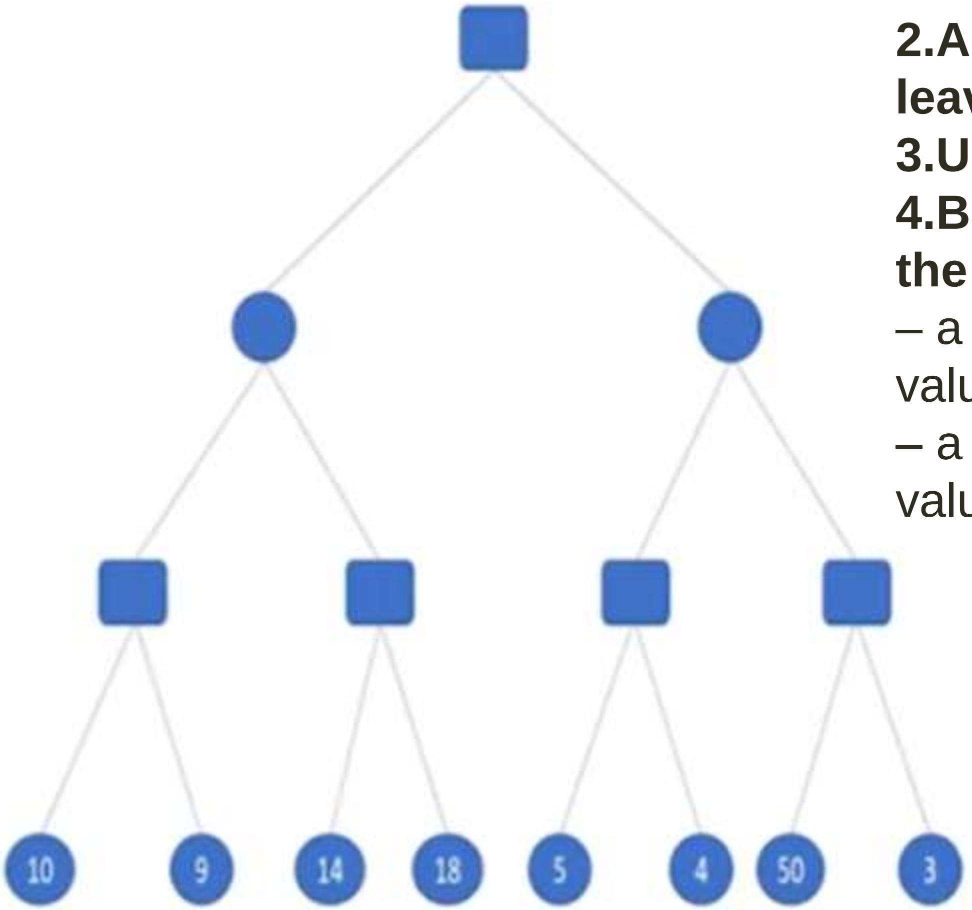
# MINMAX ALGORITHM

The MinMax algorithm is used in two-player games to determine the best strategy. It assumes that both players act optimally: one seeks to maximize their own gains, while the other seeks to minimize their opponent's gains.

How it works:

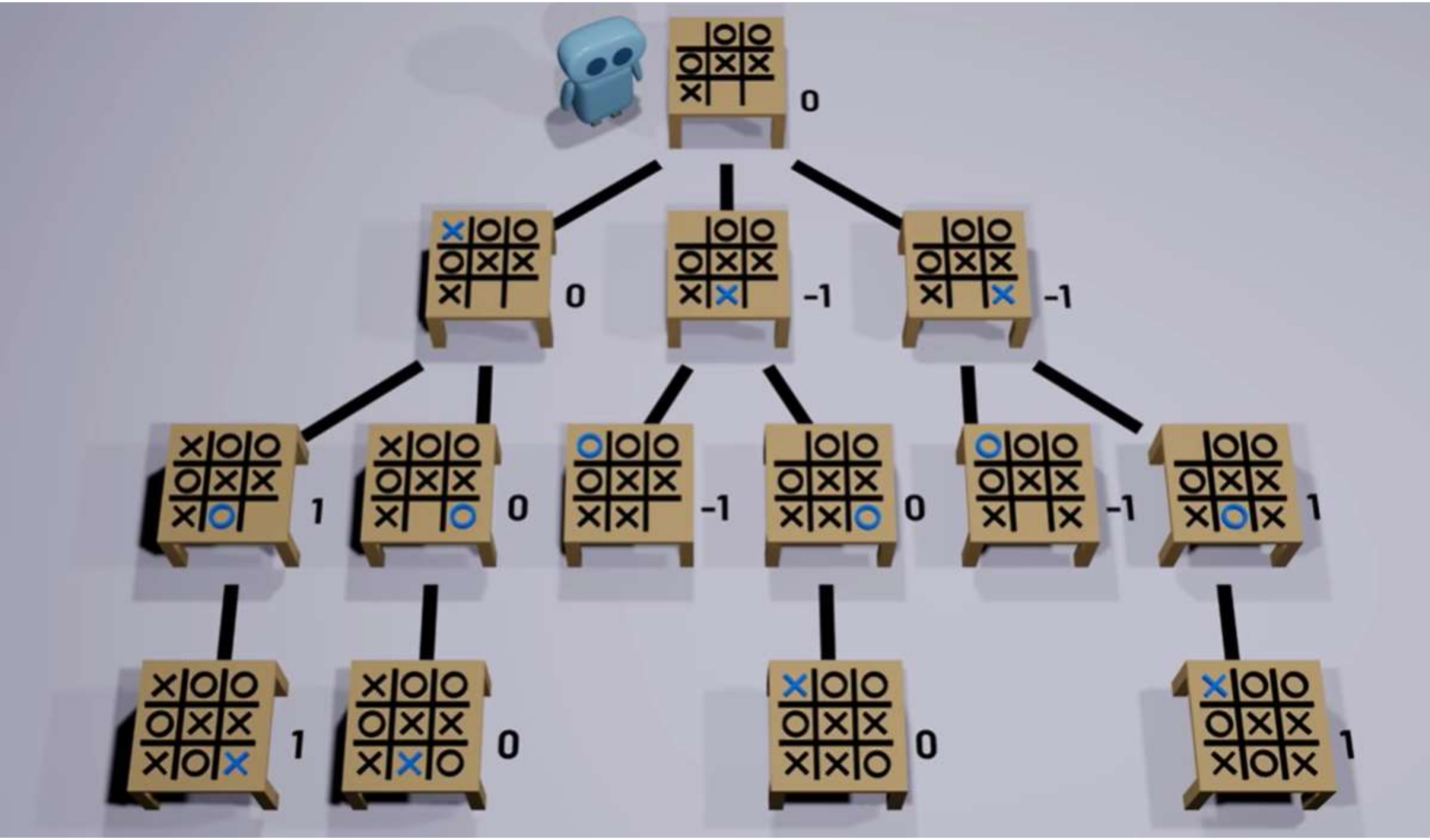
- The nodes of the game tree represent the possible states.
- The leaves of the tree contain the final scores.
- The maximizing player chooses the highest score among the children of a node.
- The minimizing player chooses the lowest score.

# EXAMPLE

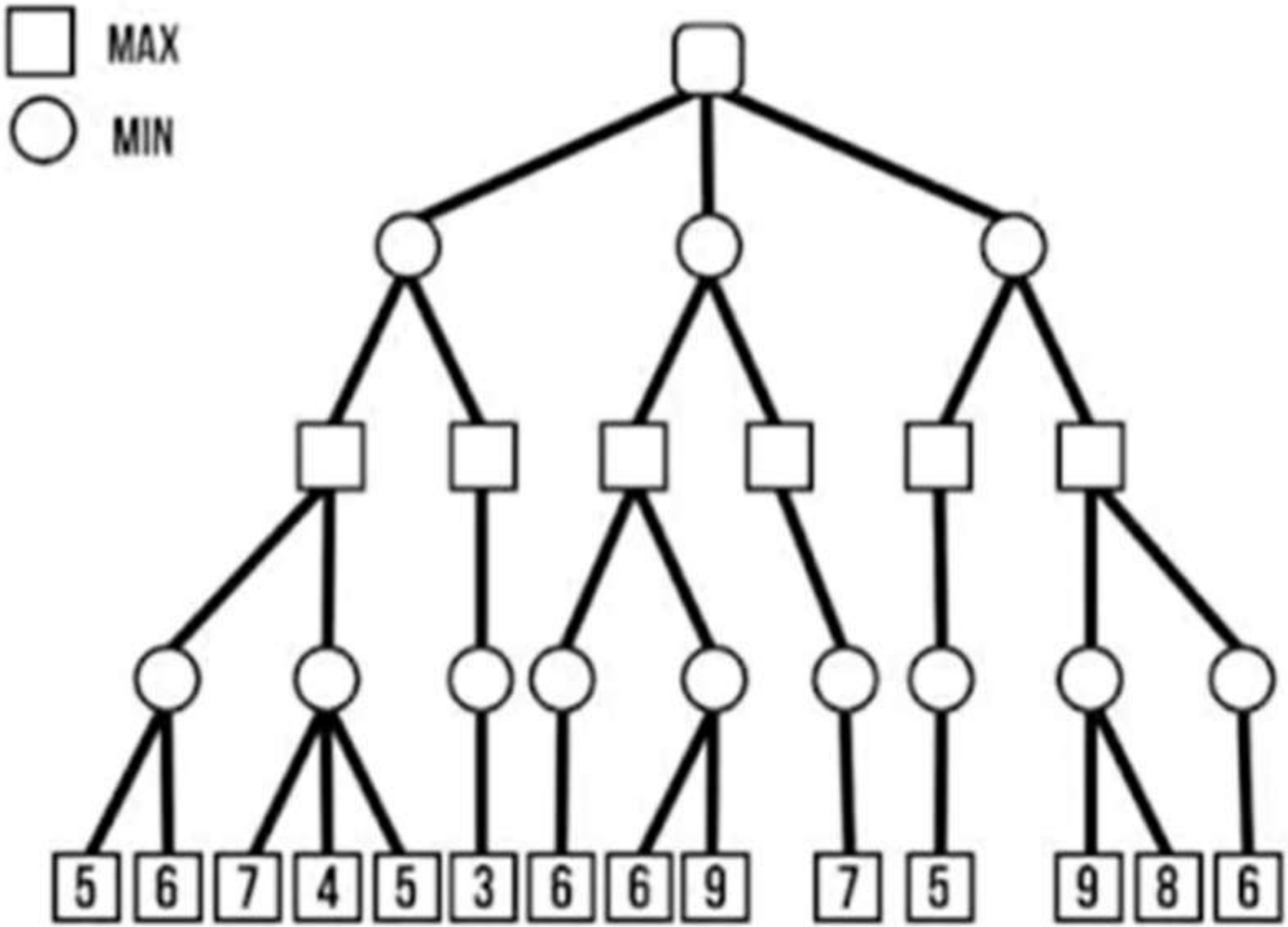



**1.Generate the whole game tree to leaves**
**2.Apply utility (payoff) function to leaves**
**3.Uses DFS for expanding the tree**
**4.Back-up values from leaves toward the root:**
– a Max node computes the maximum value from its child values
– a Min node computes the minimum value from its child values

# EXEMPLE : ALGORITHME MINMAX



# EXEMPLE : ALGORITHME MINMAX



# ALPHA-BETA ALGORITHM

The Alpha-Beta algorithm is an optimization of the MinMax algorithm. It prunes unnecessary branches in the game tree, thus reducing the number of calculations required.

Alpha: The maximum value that the maximizing player can guarantee so far.

Beta: The minimum value that the minimizing player can guarantee so far.

Principle:

- If a branch offers a value lower than Alpha or higher than Beta, it is ignored.

- This reduces the number of nodes to explore.